XII Crisis and reEvolution of the foundations of mathematics

The antinomies and paradoxical properties of Georg Cantors set theory led to a crisis of the foundations of mathematics at the end of the 19th century. The axiomatic system of Zermelo and Fraenkel, ZFC, was intended to create foundations free of contradictions. The vast majority of mathematicians and logicians assumed that this had been achieved. **In the book „Nichts“ it is shown that crucial foundations are based on metaphysical assumptions that contradict reality and are therefore false.**
The crisis was not overcome by ZFC, but deepened. The author also counts Gödels demand of incompleteness of the theory of natural numbers and the claim that the syntax of mathematical logic is meaningless among the symptoms of the crisis.
However, the phenomenal development of applied and pure mathematics is not affected by this. But an adequate evolution of the foundations has not taken place, and the banishment of "nothing" from mathematics and logic is closely related to this.
The errors of the current foundations and their correction are shown in the following summary of Articles I - XI:

I Empirical fact : 0 is not a number, but represents "nothing"
The factual meaning of 0, "nothing", "no number" was respected for 3 millennia before it was changed into its opposite, “number“, in the 16th century due to reservations about the metaphysical 'Nothing'. This violates one of the most important axioms of mathematical logic, the 'tertium non datur'. The original meaning must be restored to define the 0. **The “number“ 0 is contradictory.** This has consequences for the transfinite numbers:

II Inconsistence of the transfinite number ω and the set ℕ
The existence of Cantors transfinite numbers presupposes 0 as a ”natural number” in the definition of the ZFC infinity axiom. The necessary recourse to the original meaning of 0, "nothing",“no number“, proves the inconsistence of transfinite numbers.
Cantors justification of transfinite numbers and sets occured in a different way, it is not axiomatic. The errors are discussed in Article III.
David Hilbert, the most important mathematician of his time, confirmed that the transfinite contradicts reality and is only an idea, but insists that the mathematical theory is consistent. But **an idea that contradicts reality is necessarily false.** This confirms Henri Poincaré, who criticised that future generations would regard the transfinite as a disease from which mathematics had recovered.

III Cantors errors in his doctrine of the transfinite
- Cantors errors above all come from his **incorrect justification of the 1st transfinite cardinal number ℵo.** He assumes the bijection of a set with the cardinal number ℵo with a set that contains a further element, and accordingly has the cardinal number ℵo + 1. The alleged bijection - that it does not exist is demonstrated in Article III - would justify the equation ℵo = ℵo + 1. In contrast to this, however, Cantor demands the equation ω ≠ ω+1 due to the further, larger element. In the finite, cardinal and ordinal numbers are always equal; in the transfinite, an antagonistic difference is falsely asserted:
(1) ℵo = ℵo + 1 but ω ≠ ω+1.
- The **continuum hypothesis** is justified by Cantor as follows:
Not only does ℵo = ℵo + 1 apply, but the equality of many other cardinal numbers is derived. By adding 1 on both sides, we obtain ℵo + 1 = ℵo + 2. The sequence of equal cardinal numbers continues until $ℵo^n$:
(2) ℵo = ℵo + 1 = ℵo + 2 = ... $ℵo^n$
The rational numbers also have the cardinal number ℵo.
But for the cardinal number of the set of real numbers, $2^{ℵo}$, Cantor demands by his 2nd diagonal argument:

(3) $2^{\aleph_0} > \aleph_0$
The conjecture that there are no further cardinal numbers between $\aleph_0$ and $2^{\aleph_0}$ is known as the 'continuum hypothesis'. However, the formula used by Cantor to calculate the real numbers (see Article III) shows that $2^{\aleph_0}$ is not the cardinal number of the real numbers, but that of the rational numbers. Therefore applies:
(4) $2^{\aleph_0} = \aleph_0$
Cantor's 2nd diagonal argument, which requires (3), is false, the continuum hypothesis is obsolete.
- Cantor demands the **equality of sets of points of different geometric objects (** arbitrarily small distance, infinite distance and area, infinite-dimensional space). The equality is supposed to hold since the cardinal numbers of the sets of points of these objects are equal. Cantor therefore assumes that the cardinal numbers can be mapped onto points.
However, in his justification of $\aleph_0 = \aleph_0 + 1$, he already requires that, despite this equality, there is another element, i.e. another point. This is also confirmed by $\omega+1 > \omega$. It is not the equality of the cardinal numbers that determines the sets of points, but the unequal ordinal numbers. The claimed equality of sets of points does not exist.
- For Cantor, the **equivalence of sets and subsets** is the most characteristic property of the transfinite, in contradiction to Euclids 5th axiom, "The whole is greater than the part". In fact, equivalence already applies to finite sets. Different spatial densities of the elements of sets and subsets are responsible for the paradox. Euclids axiom remains valid.

IV The inconsistency of the empty and the transfinite set
The existence of the empty set $\varnothing$ is axiomatically required by ZFC. This is the only way to justify the transfinite set. $\varnothing$ is defined as a set without elements, but is itself an element. $\varnothing$ can be united with the elements of any set and then enlarges the set.
The empty set is reviewed empirically. A set without apples, for example, which would enlarge a set of apples, is a contradiction in terms, the set remains the same. This applies to any elements. **The existence of the empty set is empirically disproved. This means that the existence of the transfinite set is also contradictory.**

V Comprehensive meaning of ∞ and elimination of contradictions
The infinite can only be described by the symbol ∞ after the transfinite has been refuted. The following applies to the potentially infinite, permanently increasing length:
(1) $l_{inf} = \infty$
linf is mapped to the infinite natural number.
(2) $n_{inf} = \infty$
This characterises the absolute value, but the number of length units or natural numbers is also described by ∞. However, depending on the spacial density of the elements, there are many infinite sequences of natural numbers 1, 2, 3, ... n .... on the number line. Euclid solved the problem of comparability. "Quantities are called commensurable if they have a common measure". He demanded the **determination of standard quantities.**
linf is divided into the infinite sequence of standard length units lSi. The characters < > stand for the ever-increasing number of standard lengths, the following applies :
(3) $< l_{Si} > = \infty$
The standard lengths are mapped to standard natural numbers n_{Si} :
(4) $< n_{Si} > = \infty$
Subdivision into non-standard lengths also **results in multiples or parts of** ∞. For distances l, for which e.g. l = ls / n applies, you get :
(5) $< l_S / n > = n \cdot \infty \mid n \cdot \infty > \infty$
This result is consistent with the fact that the potentially infinite is ultimately finite, i.e. the rules of the finite must apply. At present, $n \cdot \infty = \infty$ is contradictorily demanded and, for example, $\infty + \infty = \infty$. In fact :
(6) $\infty + \infty = 2\,\infty$

VI Planck units disprove converging infinite sequences and limits
Only finite converging sequences can be justified

Since Euclid, the infinite subdivision of distances has been assumed with the consequence of mapping to real numbers with an infinite number of digits. David Hilbert adopted this assumption in 1903 in his *Foundation of Geometry*, and it is still part of the mandatory system of geometry today. Max Planck, on the other hand, demanded units such as the Planck length in 1899, which cannot be undercut. Hilbert conceded in 1926 that the **infinite subdivision contradicts reality,** but did not draw consequences for mathematics from this. The Planck length prohibits the infinite subdivision of the distance and thus also the mapping to numbers with an infinite number of digits. A limit of infinite converging sequences of digits therefore does not exist; it is replaced by the limitation of finite converging sequences by a last digit z. Irrational numbers and rational numbers with an infinite sequence of digits cannot be justified to exist. For example, (1) can no longer be demanded, (2) applies

(1) $\lim_{n \to \infty} 1.414213 \ldots\ldots\ldots\ldots\ldots\ldots = \sqrt{2}$

(2) $\sqrt{2} = 1{,}414213 \ldots\ldots\ldots z$

z is reached when the geometric mapping is smaller than the Planck length.

Analysis, differential and integral calculus, was developed by Augustin-Louis Cauchy on the basis of the limit value, which also assumes infinite subdivision. Potentially infinite sequences of numbers with increasingly smaller differences between neighbouring members, Cauchy sequences of real numbers, are required, which converge for $n \to \infty$ in a limit value, the limes, e.g. $\sqrt{2}$ according to (1).

As a consequence of the Planck length, the limit is replaced by the limited finite sequence, see e.g. (2). This simplifies analysis considerably.

The method of limiting by finite sequences of digits also corresponds to the real handling, which does not allow infinite sequences of digits.

Article VII Refutation of the undecidability of Gödels proposition

In 1931, Kurt Gödel constructed proposition G of the theory of natural numbers which states about itself that no axiomatically founded sequence of formulae exists which could prove the proposition with the Gödel number $\ulcorner G \urcorner$:

(1) $G = \neg (\exists x: x \vdash \ulcorner G \urcorner)$

For this theorem, in turn, Gödel demands in his 1st incompleteness theorem that neither a proof nor a refutation by a sequence exists, although it is true. This statement is correct, but it does not prove that the proposition G could not be decided after all.

The true meaning of 0 according to Article I, the sign for "nothing", opens up new possibilities of proof. "Nothing" and "non-existence" are equivalent. Propositions about "non-existence" can be proved by equivalent propositions about "nothing". **The new axiom of "nothing" of proof** (2) is defined. The existence of the sign $\mathbb{Z} = 0$ for an empty place, "nothing", is the premise of the proof. Under this prerequisite, no sequence Γ exists that could prove a proposition φnp that cannot be proved by a sequence.

(2) $(\exists \mathbb{Z}: \mathbb{Z} = 0) \vdash \neg (\exists \Gamma: \Gamma \vdash \varphi_{np})$

The axiom is applied to Gödel's proposition G and mapped to Gödel numbers. The variable sign $\mathbb{Z}$ and the variable Γ become the number variables x and x > 1.

According to Gödel, 0 has the Gödel number 1. This gives us:

(3) $(\exists x: x = 1) \vdash \neg (\exists x > 1: x > 1 \vdash \ulcorner G \urcorner)$

The right-hand side of ' $\vdash$' is Gödel's proposition (1). It therefore applies:

(4) $(\exists x: x = 1) \vdash G$

The negation of G cannot be proved, as can be seen from (3).

Gödel's proposition, which is based on the "non-existence" of his proof, **is decided by "nothing" of proof.** The theory of natural numbers is complete, the axiomatic system that Gödel presupposed is incomplete. It is completed by the new 'axiom of "nothing" of the theory of proof'.

VIII Fiction of the meaninglessness of the syntax of mathematical logic
David Hilbert saw a "completely satisfying way" to escape the antinomies of set theory. Through his formalism, he demands the meaninglessness of the syntax of mathematical logic. Semantics should then only allow contradiction-free interpretations. The overview of syntactic signs, formulae and rules shows that interpretation is already inherent. **The alleged meaninglessness is an unreal fiction that does not protect against inconsistency.** Russell's antinomy still exists, as does the inconsistency of the transfinite. In fact, the significance of syntax lies in the fact that it fulfils every semantic interpretation in the highest abstraction.

IX Nothing, "nothing", nothing, "nothing"
A discussion expressing uncertainty and doubt about the existence or non-existence of “nothing“ was decided by Plato in the Sophistes: "The non-existent was and is non-existent and is to be counted as a concept among the many existent things. **"Nothing" does not exist, but the concept of "Nothing" does**. The metaphysical "Nothing" represents absolute non-existence, while the mathematical "nothing" means something completely different. The difference to nothing without inverted commas must be clarified
There is nothing. This means that something that exists is not present at a certain point. Digits and numbers exist, but they are not present in an empty place. Therefore for a gap in a sequence of digits it is true that there is nothing, **the marking of this empty place, the 0, represents the "nothing"**. The 0 is currently wrongly defined as a number, in fact it represents an empty place, "nothing", "no number".
The 0 also represents the "nothing" of the theory of proof. The sign for an empty place indicates that there is no sequence that could prove a proposition.
The "nothing" of sets is represented by the sign ∅. ∅ is currently wrongly defined as the "empty set", which can be added as an element of a set and enlarges it. In fact, the signrepresents "nothing" of sets, "no set", which cannot enlarge a set. The "empty set" contradicts reality, it is contradictory.

X Resolution of the antinomies
Only the two most important antinomies are considered here, Russell's and the 'liar'.
- Russells antinomy, which questions the core of set theory, the concept of set, sparked the so-called fundamental crisis in mathematics at the beginning of the 20th century. Russell distinguishes between sets that contain themselves and sets that do not contain themselves. The bone of contention is the set R, the set of all sets that do not contain themselves.
If R does not contain itself, this contradicts the premise that a l l sets that do not contain themselves should be summarised. If R contains itself, this violates the condition that o n l y sets that do not contain themselves should be considered.
But if sets contained themselves, the primal elements would be counted twice. From this follows :
Sets cannot contain themselves, not even R. **This resolves Russells antinomy.**
- The liar's antinomy states: Epimenides, the Cretan, says: **"All Cretans lie".**
It is not possible to decide whether the Cretan is telling the truth or whether he is lying.
The Cretan's proposition contains a hidden variable, **the 'lie'. If this is specified,** the variable is bound, then **it can also be decided** whether all Cretans are lying it or not.
Not all linguistic propositions that contain a hidden variable such as 'lie' are undecidable. Only the combination with self-reference implies undecidability in analogy to Gödels theorem.

XI Crisis in the thinking of the theorists of the foundations of mathematics
- The 0 has been seen as a number since the 16th century. In the previous 3 millennia, however, the fact that 0 denotes an empty place, i.e. represents "nothing", "no digit" and "no number", was premised to be true. The reversal of the meaning of 0 to be a “number“ violates the ' tertium non datur' and is therefore contradictory. It goes back to philosophical and religious reservations about the metaphysical 'Nothing' with which 0 was confused.
- An allegedly consistent theory of the transfinite is postulated as part of the foundations of

mathematics, although it was already confirmed a century ago that it contradicts reality. This way of thinking by theorists of foundations violates the criteria of scientificity. A theory that contradicts reality is necessarily and demonstrably false.
- The insistence on the infinite subdivision of distances in contradiction to Planck also shows that physics, i.e. reality, is disregarded in the humanities of mathematics.
- The empty set, which is axiomatically postulated to justify the transfinite, also fails in the face of reality.
- The failure to resolve the antinomies led to the fiction of the meaninglessness of the syntax of mathematical logic, whereby the antinomies were to be avoided; these however persit. In fact, the meaning of the syntax consists in the fact that it fulfils every semantic interpretation in the highest abstraction.
- The refusal of theorists to recognise the refutation of the incompleteness of the theory of natural numbers by the existence of the "nothing" of proof also demonstrates a crisis of thought.
- **The decades-long crisis in the foundations of mathematics that emerged before the 20th century was not overcome by the measures taken to overcome it by means of the axiomatic system ZFC, but rather was intensified. The banishment of "nothing" from mathematics and logic is recognised as a major cause of the crisis, which is overcome by restituting "nothing". A re-evolution of the foundations is realised.**