VIII Fiction of the insignificance of the syntax of mathematical logic[1]

Dec 2022. David Hilbert saw a “completely satisfying way” to escape the antinomies of set theory. Through his formalism he calls for the insignificance of the syntax of mathematical logic. Semantics should then only allow consistent interpretations. The overview of the syntactic signs, formulas and rules shows that the interpretation is already inherent. The alleged insignificance is an unreal fiction, which also did not protect against inconsistency.

1. Introduction

Set theory and mathematical logic form the two foundations of mathematics First order logic is the most important pillar of mathematical logic, it is the language in which set theory is discussed. The antinomies of Cantor's set theory provoked a fundamental crisis in mathematics, which also had an impact on **mathematical logic. Three important schools** emerged, the **logicism** formed by Gottlob Frege and Bertrand Russell, the **intuitionism** represented primarily by Luitzen Brouwer, and the **formalism** demanded by David Hilbert. His reasoning ultimately prevailed. The strict separation of a formal, meaningless syntax and its interpretation by semantics is advocated.

2. Hilbert's formalism

Hilbert describes a "completely satisfying way to escape from the paradoxes[2] "As we have seen, the abstract operation with general concepts and contents has turned out to be inadequate and uncertain." As a prerequisite for **a new foundation of mathematics** he names "**the sign, ... which has no meaning”.** He also states that the **formulas of the logical calculus[3] have no meaning** in themselves and **rules are built by purely formal processes.** Hilbert expressly distances himself from Frege, who presupposes sense and meaning. He then essentially defines the syntax of today's first order logic (see 3.). A predicate in this context is a formula with a free variable. In the 2nd step, the semantics "take the considerations of content ..... to a higher level ..... at the same time, in mathematics, a strict and systematic separation between the formulas and formal proofs on the one hand and the considerations of content on the other hand is possible." **Hilbert states that the foundation of mathematics is ensured purely syntactically, free from semantic interpretation.**

3. Outline of the syntax of first order logic

In order to be able to judge the formalism with the postulate of the insignificance of the syntax, its main features are outlined.

3.1. signs

logical signs:

variable x, x_0, x_1, ….
propositional combinations ¬ (not), ∧ (and), ∨ (or), → (implies), ↔ (equivalent)
existence quantifier ∃ (there exists)
the equal sign =

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1 The topic including the bibliographical references are part of the book by Gert Treiber, *" Nichts", Krise und reEvolution der Grundlagen der Mathematik,* Cuvillier Verlag, 2020

2 Today: antinomies

3 Theory of mathematical logic, first orde predicate calculus

non-logical signs
relation sign >, <
function sign +, ·
individual constants c, d, e, ...

3.2. Terms, formulas and rules

Terms can be formed with this alphabet. Anything that can be on one side of an equation or relation suffices as a term. Linking terms with a function sign results in a new term. If e.g. x and y are terms, then x + y is also a term. Terms are linked to formulas using relation signs.
The syntactically permissible formal rules are defined as follows:

1. If φ is a formula, then so is $\neg \varphi$.
2. If φ and ψ are formulas, then also $\varphi \vee \psi$, $\varphi \wedge \psi$, $\varphi \rightarrow \psi$, $\varphi \leftrightarrow \psi$
3. If φ is a formula and x is a variable, then $\exists x \varphi$ is also a formula.
4. s = t for arbitrary terms s, t.
5. $R (t_1, \ldots\ldots t_n)$ for every relation sign R and arbitrary terms $t_1, \ldots. t_n$
   An example for $R (t_1, t_2)$ is e.g. e.g. $n < m$.
6. These are all the formulas.

4. Criticism of formalism and refutation of insignificance of syntax

Critics rejected the "completely meaningless intrinsical mathematics". "Don't let yourself be deceived by the complete lack of content in Hilbertian mathematics. ….. Signs that mean nothing are signs stripped of their pointing function, i.e. no signs at all.” Other critics have expressed similar sentiments.
Semiotics as a superordinate science also contradicts Hilbert's approach to meaningless signs. They point to something, they 'mean'. Aristotle, who was the first to describe semiotics, already premised this assumption and it also applies afterwards as to Charles Sanders Peirce.
That from the signs 3.1. exactly and only let the formulas and rules 3.2 are formed, if everything is supposed to be meaningless, is an unreal fiction. In fact, the formulas and rules can only be formed if the sense and meaning of the signs are inherently presupposed, as is demonstrated in the book “Nichts”.
Nevertheless, Hilbert's formalism has prevailed. **However, the problem is not the differentiation between syntax and semantics, but rather the supposed insignificance and independence of syntax from reality.** Hilbert's motive for demanding irrelevance was to escape the antinomies. But this does not resolve a single antinomy, above all the source of the foundational crisis, Russell's antinomy, remains unresolved and inconsistency of the axioms was not avoided.
The effects can hardly be underestimated.
Mathematicians and logicians exclude themselves from the intuitive perception of interrelation and evade the purgatory of possible falsification by reality. Identifying inconsistencies is made extremely difficult. The author's question to a chair holder what "actually infinite" and "transfinite" mean in today's mathematics was answered with "I don't know what actually infinite means, I don't know what transfinite means". This expressed the senselessness of the question about the meaning of something meaningless.
The fatal consequences of assuming meaningless signs, formulas and rules are thus demonstrated. Anyone who presupposes insignificance must reckon with the fact that nonsense can result. Hilbert confuses abstraction from reality with independence from reality. The "refoundation of mathematics" on a meaningless syntactic formalism is a postulate that does not stand up to reality. This does not affect the author's appreciation of formal logic. But he demands that the meaning and sense of the syntax be respected. **Indeed its importance is founded in the fact that it satisfies every semantic interpretation in the highest abstraction.**