VI Planck units refute converging infinite sequences and limits [1]
Only finite converging sequences can be justified

Nov. 2022. The irrational numbers are defined by limits of potentially infinite converging sequences of rational numbers that require infinite sequences of digits of the irrational numbers. Real numbers can be generated by mapping of segments. However, the subdivision of distances is limited by the Planck length, which only allows finite digits when mapped to real numbers. Potentially infinite sequences of ever smaller differences of distances and numbers as well as limits can no longer be justified. They are replaced by finite sequences with limitation of digits
This also applies to analysis, the differential and integral calculus, the Planck-length causes limitation. Limits, $\Delta x \rightarrow 0$, $n \rightarrow \infty$ and infinitesimals do not occur. Limitation and $\Delta x = 0$, "nothing"[2], are the decisive criteria. In practice already always limitation-values are determined, the calculation must be terminated sometime.

1. Introduction

Since Euclid, the infinite subdivision of distances has been assumed, with the consequence of mapping to real numbers with infinite numbers of digits. In 1903, David Hilbert adopted this premise in his *Grundlagen der Geometrie* (*Fundamentals of Geometry)* and it is still part of the obligatory system of geometry today. **Max Planck, on the other hand, in 1899 demanded units such as the Planck length, which cannot be fallen below.** Hilbert admitted in 1926 that the infinite subdivision contradicts reality, but drew no conclusions for mathematics.
Isaac Newton assumed ever smaller infinitesimals when founding the differential and integral calculus, the analysis[3].
Analysis was rebuilt by Augustin-Louis Cauchy on the basis of the limit, also assuming infinite subdivision. This presumption turns out to be wrong on the basis of today's physics with a limitation by Planck units. Consequences are discussed in the following.

2. Rational and irrational numbers

2.1. Current theory

Rational numbers are defined as fractions of whole numbers, irrational numbers cannot be represented by such quotients. Together they build the real numbers. However, irrational numbers cannot be represented numerically. The number √2, for example, is not yet defined as a decimal fraction with a potentially infinite number of digits $n \rightarrow \infty$. Only the limit of the potentially infinite sequence of approximated values determines √2 (1).

(1) $\lim_{n \rightarrow \infty} 1.414213 \ldots\ldots\ldots\ldots = \sqrt{2}$

Since irrational numbers do not exist as numerical numbers, the question of the justification

1 The topic including the bibliographical references are part of the book by Gert Treiber, *" Nichts", Krise und reEvolution der Grundlagen der Mathematik,* Cuvillier Verlag 2020.
2 Article I Empirical fact: The 0 is not a number but represents "nothing"
3 Gottfried Wilhelm Leibniz, on the other hand, demanded constant but infinitely small infinitesimals. Meanwhile the infinitesimals dx, dy, ... only play the role of a historical notation without practical meaning.

of the limit arises. It seems justified by the geometric segment, e.g.√2. The mapping of segments and other geometric elements to numbers is a core element of mathematics since Euclid.

2.2. The consequences of the Planck-length

The Planck length excludes infinite subdivision. Under this premise the mapping to numbers shows fundamental changes in comparison to current theory. The infinite sequences of decimals therefore give way to finite sequences. The last decimal place z of minimum value corresponds to the minimum distance, the Planck length. √2 is expressed in real terms by (2), further digits can be calculated but are meaningless. **A limit of infinite converging sequences (1) does not exist; the limitation of finite converging sequences (2) takes its place.**

(2) $\sqrt{2} = 1.414213 \ldots\ldots\ldots\ldots\ldots z$

By mapping to a Planck length a minimal rational number r_{min} exists.

The geometric representation of √2 is also limited by Planck lengths, i.e. the existence of the geometrical segment √2 cannot justify the limit of irrational numbers.

Numbers whose digits are finite, can be represented as the quotient of integers. Only rational numbers with a finite number of digits can be demanded, **irrational numbers and also rational numbers with an infinite number of digits lose their justification..**

This statement seems to be contradicted by Euclid who presented a proof for the irrationality of √2, i.e. demonstrating that √2 cannot be represented by the quotient of integers.

The refutation can be found in the book "Nichts", it is only noted here that Euclid's proof assumes infinite subdivision. This statement also applies to the proof by Dedekind cuts and interval nesting.

According to current theory, the transfinite set of real numbers of the interval [0, 1] with the cardinal number $2^{\aleph_0}$ builds the continuum. This set is not countable by the natural numbers.

In reality, the geometric continuum is built by a finite sequence of Planck lengths respectively their aggregation to subsegments of the segment [0, 1]. By mapping the geometric continuum, the arithmetic continuum is built by the finite sequence of rational numbers of the interval [0, 1]. The set results after defining a standard length.

3. Analysis, differential and integral calculus

3.1. Current theory

3.1.1. The **definition of the limit** is one of the most important theorems in analysis. The introduction of a quantity ε as an element of the real numbers in a potentially infinite sequence $(a_n)_n$ is crucial.

(3) A sequence $(a_n)_n$ is called convergent to the limit a_{limit} if for all real numbers $\varepsilon > 0$ there exists a natural number m such that for all $n > m$ it holds: $|a_n - a_{limit}| < \varepsilon$.

With regard to the detailed discussion, reference is made to the book "Nichts". **The decisive issue is that potentially infinite sequences of numbers with an ever-decreasing difference between neighboring members, Cauchy sequences of real numbers, are required, that converge for $n \to \infty$ in a limit.**

3.1.2. In **differential calculus,** the gradient of the tangent of a function y = f (x) is determined. The gradient is determined as the limit of a sequence of approximated values Δy / Δx. In the process of differentiation Δx is decreased to smaller and smaller values in a potentially infinite number of steps in order to finally obtain the gradient with the **limit Δx → 0**. At the points x_i the gradient results as limit $\Delta y / \Delta x = f'(x_i)$. Thereby $\Delta y = f (x_i + \Delta x) - f (x_i)$ applies.

(4) $\lim_{\Delta x \to 0} [f (x_i + \Delta x) - f (x_i)] / \Delta x = f' (x_i)$

3.1.3. Through the **integral calculus,** the area F (x) between the function f (x) and the x-axis is determined between x = a and x = b. In practice the area is divided into n rectangles of width Δx. **The limit is determined for $n \to \infty$ and $\Delta x \to 0$:**

(5) $\lim \int^{b} f_n(x) \cdot \Delta x = F (b) - F (a)$

$n \to \infty$ a $\Delta x \to 0$

3.2. Consequences of the Planck-length

3.2.1. The limit (3) is replaced by the limitation (6). A sequence (a_n)$_n$ is said to converge to the limitation $a_{limitation}$ if the calculational difference between the last two terms, $a_z - a_{z-1}$, is equal to or smaller than r_{min} .

(6) limitation (a_n)$_n$ = $a_{limitation}$
$a_z - a_{z-1} \leq r_{min}$

3.2.2. In reality, the numbers f ' (x) of the differential calculus are already not limits that satisfiy definition (4), as demonstrated in the book “Nichts”.
In fact, the quotient Δy / Δx is developed in finite steps in such a way that a uniquely determined **limitation** Δy / Δx is determined for Δx = 0 (7). Potentially infinite sequences of calculation steps with Δx → 0 do not occur, a limit does not exist. In practice the number of digits is limited to finite values.

(7) limitation = [f (x_i + Δx) – f (x_i)] / Δx = f ' (x_i)
$\Delta x = 0$

3.3.3. For the **integral calculus,** too, a finite sequence of steps results in a clearly defined **limitation for Δx = 0.** The sum of the n rectangles of width Δx between a and b is transformed in such a way that n is eliminated and Δx = 0 can be set. n → ∞ does not occur.

(8) limitation $\sum_{a}^{b}$ (f (x_i) · Δx (i = 1,2, …. n) = F (b) – F (a)
$\Delta x = 0$

Functions are differentiable and integrable if the gradient and the integral as f (Δx) can be formulated in such a way that Δx = 0 results in a uniquely determined limitation value.
The limit, Δ x → 0, n → ∞ and infinitesimals do not occur. Potentially infinite converging sequences and limits are unreal ideas. In practice, **when concrete numerical values are calculated, already always limitation values must be determined necessarily,** the arithmetic operation has to be terminated sometime. An infinite number of operations, that would result in a numeric limit, is impossible. Potentially infinite sequences and limits are unreal ideas.
Limitation and Δx = 0, "nothing", already now are the decisive criteria of analysis.