## V Comprehensive meaning of ∞ and elimination of contradictions

Nov. 2022. The symbol ∞ can only be completely understood if a standard length is assumed. The axiom of infinity defines ∞ by the potentially infinite sequence of standard units of length on the axis of Euclidean space. When non-standard quantities are considered, counts are greater or less than ∞. This also applies to natural numbers resulting from mapping line segments. Cantor premised a single transfinite set of natural numbers, in fact many unlimited infinite sequences of these numbers exist. In contrast to previous theory, consistent calculation rules apply, e.g. $\infty + n > \infty$ and $\infty + \infty = 2\ \infty$.

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### 1 Introduction

The ZFC-axiom of infinity, that requires existence of the transfinite set ℕ and the transfinite ordinal number ω of natural numbers, is contradictory[1]. The description of infinity is thus traced back to the symbol ∞[2]. Its meaning is not yet fully explored and contradictory properties are ascribed to it. First it is realized that **many unlimited infinite sequences, iS, of natural numbers 1, 2, 3, .... n .... exist on the number line.** Cantor in contrast had premised a single transfinite set of these numbers. Three different iS are mapped.

Tab. 1 Various infinite sequences of natural numbers.

| | | | | | | |
|---|---|---|---|---|---|---|
| ( a ) | 0 | | 1 | | 2 …................. n ............................................. | |
| ( b ) | 0 | 1 | 2 | 3 | 4 …................................ | n .............................. |
| ( c ) | 0 | | | | 1 | 2.................... n ........ |

**At present, this disparity cannot be described comparatively with the sign ∞.**
**In addition, ∞ stands for the potentially infinite, i.e. ultimately finite, so the laws of the finite should apply.** In fact, however, other rules are applied, such as:
( 1 ) $\infty + n = \infty$
( 2 ) $\infty + \infty = \infty$

### 2. ∞ and standard terms

Euclid solved the problem of comparability. "Terms are called commensurable if they have a common measure". He demands the definition of standard units. These are not yet relevant for the absolute value of infinity in Fig. 1. The following applies to infinite length $l_{inf} = \infty$ ( a ). $l_{inf}$ is mapped to the same absolute value of the infinite natural number $n_{inf} = \infty$ ( b ). In order to be able to compare the various sequences in Tab. 1, $l_{inf}$ is subdivided into the iS of the standard length units $< \boldsymbol{l}_{\mathbf{Si}} > = \infty$ ( c ). The signs < > stand for the number of elements of an iS. $l_S$ and multiples of $l_S$ form an iS ( d ). $\infty l_S$ stands for the potentially infinite number of standard distances. The distances ( d ) are mapped to standard natural numbers ( e ). $\infty_S$ is the potentially infinite number of standard

1 Article II The infinite: transfinite numbers are contradictory
2 The topic including the bibliographical references are part of the book by Gert Treiber, *" Nichts", Krise und reEvolution der Grundlagen der Mathematik,* Cuvillier Verlag 2020.

natural numbers. It holds that $\infty_S = n_{inf} = \infty$. $\infty_S$ summarizes and completes the standard natural numbers. The representation of the iS of the natural numbers 1, 2, 3, ……, n, ……. that is usual today as is incomplete. Only ( e ) including $\infty_S$ represents the sequence comprehensively.

Fig. 1 ∞ and standard terms

( a ) 0——————————————————————————————————> $l_{inf} = \infty$
( b ) 0——————————————————————————————————> $n_{inf} = \infty$
( c ) 0<——$l_{S1}$——|——$l_{S2}$——| ..................... $l_{Sn}$——| .......................................... > $< l_{Si} > = \infty$
( d ) 0 $1l_S$ $2l_S$ $nl_S$ $\infty l_S$ $\infty l_S = l_{inf}$
( e ) 0 $1_S$ $2_S$ ............................... $n_S$ ....................................... $\infty_S$ $\infty_S = n_{inf}$

**3. Axiom of infinity and proposition of number ∞**

The explanations of section 2. are formally defined.
**∞ is the potentially infinite length $l_{inf}$ of the axis of Euclidean space at a time t.**
**( 3 ) Definition: $\infty = l_{inf}$**
The **axiom of infinity** establishes the existence of length $l_{inf} = \infty$ which is greater than any finite set $\{ l_{Si} \}$ of standard lengths. The amount of $l_S$ can be set arbitrarily.
**( 4 ) Axiom: $\exists\, l_{inf} : l_{inf} = \infty : ( \forall \{ l_{Si} \} : l_{inf} > \{ l_{Si} \} )$** ( i = 1, 2, 3, …... , n, ….. )
The potentially infinite natural number ∞ is generated by **mapping of $l_{inf}$ to $n_{inf}$ :**
( 5 ) Definition: $l_{inf} = \infty \mapsto n_{inf} = \infty$
The **proposition of the number** ∞ establishes the existence of the potentially infinite number $n_{inf} = \infty$, which is larger than any finite standard natural number $n_S$. The $n_S$ are generated by mapping segments $nl_S$.
**( 6 ) Proposition: $\exists\, n_{inf} : n_{inf} = \infty : ( \forall n_S : \infty > n_S )$ ( n = 1, 2, 3, …... , n, ……. )**

**4. ∞ and non-standard terms**

Although only a single potential infinity is defined, there are many ways of dividing it into elements of different density d ( Tab. 1 ). Density is defined by the set of units per standard unit of lengths or natural numbers.
( 7 ) $d = \{ l_i \} / l_S = \{ n_i \} / n_S$
**Depending on the density of the elements of an iS, their number is equal to, greater than, or less than ∞.** In Fig. 2 the numbers of the sequence ( a ) of Table 1 are defined as standard natural numbers, so that $< n_{Si} > = \infty$ applies. Due to higher density, sequence $< n_i >$ ( b ) owns ordinalnumber 2 ∞, for the sequence ( c ) we have $< n_i > = \infty/2$.

Fig. 2 Standard and non-standard iS of natural numbers

( a ) 0< $1_S$ $2_S$…..................$n_S$.............................................> $< n_{Si} > = \infty$
( b ) 0< 1 2 3 4 …................................n................................> $< n_i > = 2\,\infty$
( c ) 0< 1 2...................n...........> $< n_i > = \infty/2$

**Cantor assumed only a single transfinite set of the natural numbers. In contrast, there are many iS of natural numbers.**
**However, the density of the elements cannot increase indefinitely. The Planck units cannot be fallen below.** Converging infinite sequences and limits do not exist, but limitation of finite

converging sequences.by Planck units applies. The fundamental implications are discussed in Article VI.

## 5. Consistent representation and calculation rules for ∞

**The definition of ∞ as the count of the iS of standard natural numbers allows consistent calculations with ∞.** Considering standard numbers in the plane, ( 8 ) and ( 9 ) result in contrast to ( 1 ) and ( 2 ):

**( 8 )** $\infty + n > \infty$

**( 9 )** $\infty + \infty = 2\,\infty$

Either on the axis of Euclidean space there are numbers that are greater than ∞, if a **point in time t*, later than t** as assumed in ( 3 ), applies.

( 10 ) $\infty^* >= \infty$

**Numbers of iS smaller than** ∞ exist, if the counting of the elements does not start with 1 ( 11 ), but only with n + 1 ( 12 ).

( 11 ) $< 1, 2, 3, \ldots\ldots n, \ldots\ldots , \infty > = \infty$

( 12 ) $< n + 1, \ldots\ldots , \infty > = \infty - n$

A **sequence of different numbers including** ∞ can be justified for the 3-dimensional Euclidean space as well as a higher-dimensional mathematical space:

**( 13 )** $\infty + 1, \infty + 2, \ldots\ldots 2\infty , 2\infty + 1, \ldots\ldots \infty^2, \ldots\ldots \infty^3, \ldots\ldots \infty^n, \ldots\ldots \infty^\infty$

In ( 13 ) only standard numbers were taken as a basis. The consideration of **non-standard numbers** opens up a **variety of other numbers** including ∞, which will not be discussed further.