

## V Comprehensive meaning of $\infty$ and elimination of contradictions

Nov. 2022. The symbol  $\infty$  can only be completely understood if a standard length is assumed. The axiom of infinity defines  $\infty$  by the potentially infinite sequence of standard units of length on the axis of Euclidean space. When non-standard quantities are considered, counts are greater or less than  $\infty$ . This also applies to natural numbers resulting from mapping line segments. Cantor premised a single transfinite set of natural numbers, in fact many unlimited infinite sequences of these numbers exist. In contrast to previous theory, consistent calculation rules apply, e.g.  $\infty + n > \infty$  and  $\infty + \infty = 2 \infty$ .

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### 1 Introduction

The ZFC-axiom of infinity, that requires existence of the transfinite set  $\mathbb{N}$  and the transfinite ordinal number  $\omega$  of natural numbers, is contradictory<sup>1</sup>. The description of infinity is thus traced back to the symbol  $\infty$ <sup>2</sup>. Its meaning is not yet fully explored and contradictory properties are ascribed to it. First it is realized that **many unlimited infinite sequences, iS, of natural numbers 1, 2, 3, .... n .... exist on the number line**. Cantor in contrast had premised a single transfinite set of these numbers. Three different iS are mapped.

Tab. 1 Various infinite sequences of natural numbers.

( a ) 0	1	2 .....	n .....
( b ) 0	1	2	3 4 .....
( c ) 0		1	2..... n .....

**At present, this disparity cannot be described comparatively with the sign  $\infty$ .**

**In addition,  $\infty$  stands for the potentially infinite, i.e. ultimately finite, so the laws of the finite should apply.** In fact, however, other rules are applied, such as:

- ( 1 )  $\infty + n = \infty$
- ( 2 )  $\infty + \infty = \infty$

### 2. $\infty$ and standard terms

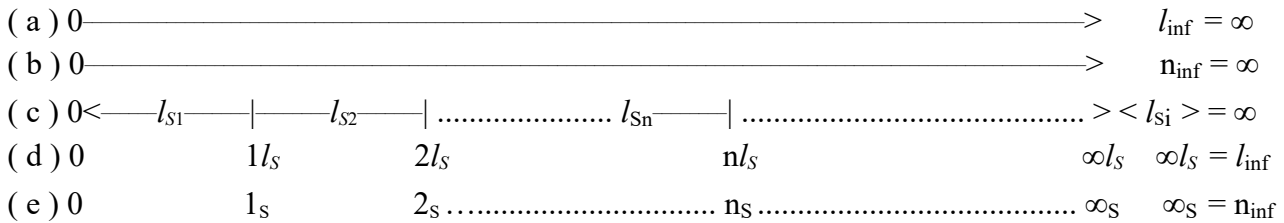
Euclid solved the problem of comparability. "Terms are called commensurable if they have a common measure". He demands the definition of standard units. These are not yet relevant for the absolute value of infinity in Fig. 1. The following applies to infinite length  $l_{\text{inf}} = \infty$  ( a ).  $l_{\text{inf}}$  is mapped to the same absolute value of the infinite natural number  $n_{\text{inf}} = \infty$  ( b ). In order to be able to compare the various sequences in Tab. 1,  $l_{\text{inf}}$  is subdivided into the iS of the standard length units  $< l_{\text{si}} > = \infty$  ( c ). The signs  $< >$  stand for the number of elements of an iS.  $l_{\text{s}}$  and multiples of  $l_{\text{s}}$  form an iS ( d ).  $\infty l_{\text{s}}$  stands for the potentially infinite number of standard distances. The distances ( d ) are mapped to standard natural numbers ( e ).  $\infty_{\text{s}}$  is the potentially infinite number of standard

1 Article II The infinite: transfinite numbers are contradictory

2 The topic including the bibliographical references are part of the book by Gert Treiber, " *Nichts*", *Krise und reEvolution der Grundlagen der Mathematik*, Cuvillier Verlag 2020.

natural numbers. It holds that  $\infty_S = n_{\text{inf}} = \infty$ .  $\infty_S$  summarizes and completes the standard natural numbers. The representation of the iS of the natural numbers 1, 2, 3, ....., n, ..... that is usual today as is incomplete. Only ( e ) including  $\infty_S$  represents the sequence comprehensively.

Fig. 1  $\infty$  and standard terms



### 3. Axiom of infinity and proposition of number $\infty$

The explanations of section 2. are formally defined.

$\infty$  is the potentially infinite length  $l_{\text{inf}}$  of the axis of Euclidean space at a time t.

( 3 ) Definition:  $\infty = l_{\text{inf}}$

The **axiom of infinity** establishes the existence of length  $l_{\text{inf}} = \infty$  which is greater than any finite set  $\{ l_{si} \}$  of standard lengths. The amount of  $l_s$  can be set arbitrarily.

( 4 ) Axiom:  $\exists l_{\text{inf}} : l_{\text{inf}} = \infty : ( \forall \{ l_{si} \} : l_{\text{inf}} > \{ l_{si} \} ) ( i = 1, 2, 3, \dots, n, \dots )$

The potentially infinite natural number  $\infty$  is generated by **mapping of  $l_{\text{inf}}$  to  $n_{\text{inf}}$**  :

( 5 ) Definition:  $l_{\text{inf}} = \infty \mapsto n_{\text{inf}} = \infty$

The **proposition of the number  $\infty$**  establishes the existence of the potentially infinite number  $n_{\text{inf}} = \infty$ , which is larger than any finite standard natural number  $n_s$ . The  $n_s$  are generated by mapping segments  $n/l_s$ .

( 6 ) Proposition:  $\exists n_{\text{inf}} : n_{\text{inf}} = \infty : ( \forall n_s : \infty > n_s ) ( n = 1, 2, 3, \dots, n, \dots )$

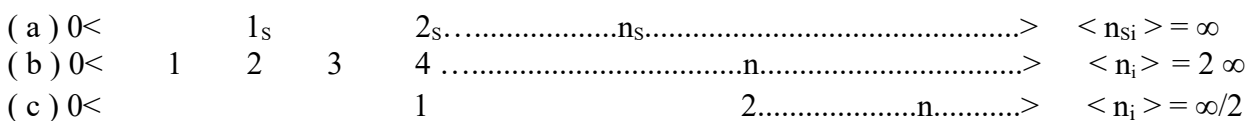
### 4. $\infty$ and non-standard terms

Although only a single potential infinity is defined, there are many ways of dividing it into elements of different density d ( Tab. 1 ). Density is defined by the set of units per standard unit of lengths or natural numbers.

( 7 )  $d = \{ l_i \} / l_s = \{ n_i \} / n_s$

**Depending on the density of the elements of an iS, their number is equal to, greater than, or less than  $\infty$ .** In Fig. 2 the numbers of the sequence ( a ) of Table 1 are defined as standard natural numbers, so that  $< n_{si} > = \infty$  applies. Due to higher density, sequence  $< n_i > ( b )$  owns ordinalnumber  $2 \infty$ , for the sequence ( c ) we have  $< n_i > = \infty/2$ .

Fig. 2 Standard and non-standard iS of natural numbers



**Cantor assumed only a single transfinite set of the natural numbers. In contrast, there are many iS of natural numbers.**

**However, the density of the elements cannot increase indefinitely. The Planck units cannot be fallen below.** Converging infinite sequences and limits do not exist, but limitation of finite

converging sequences by Planck units applies. The fundamental implications are discussed in Article VI.

## 5. Consistent representation and calculation rules for $\infty$

**The definition of  $\infty$  as the count of the iS of standard natural numbers allows consistent calculations with  $\infty$ .** Considering standard numbers in the plane, ( 8 ) and ( 9 ) result in contrast to ( 1 ) and ( 2 ):

$$( 8 ) \quad \infty + n > \infty$$

$$( 9 ) \quad \infty + \infty = 2 \infty$$

Either on the axis of Euclidean space there are numbers that are greater than  $\infty$ , if a **point in time  $t^*$ , later than  $t$**  as assumed in ( 3 ), applies.

$$( 10 ) \quad \infty^* > \infty$$

**Numbers of iS smaller than  $\infty$**  exist, if the counting of the elements does not start with 1 ( 11 ), but only with  $n + 1$  ( 12 ).

$$( 11 ) < 1, 2, 3, \dots, n, \dots, \infty > = \infty$$

$$( 12 ) < n + 1, \dots, \infty > = \infty - n$$

**A sequence of different numbers including  $\infty$**  can be justified for the 3-dimensional Euclidean space as well as a higher-dimensional mathematical space:

$$( 13 ) \quad \infty + 1, \infty + 2, \dots, 2\infty, 2\infty + 1, \dots, \infty^2, \dots, \infty^3, \dots, \infty^n, \dots, \infty^\infty$$

In ( 13 ) only standard numbers were taken as a basis. The consideration of **non-standard numbers** opens up a **variety of other numbers** including  $\infty$ , which will not be discussed further.