# IX Nothing, "Nothing", nothing, "nothing"

Dec 2022. The confusion surrounding the meaning of 'nothing' by upper and lower case, with or without quotation marks is resolved in the article on hand. The definition of 0 as "number" and  $\varnothing$  as "empty set", contrary to the true meaning "nothing", implied the wrong axiomatic postulate of transfinite numbers and sets. The ban on "nothing" that began in the 16th century also deprived mathematics of a crucial potential of proof. Equivalent propositions about "nothing" and "non-existence" prove each other. The "non-existence" of a proof is proved by the existence of "nothing" of proof theory. This statement leads to the proof of Gödel's supposedly unprovable theorem and the exceedance of the incompleteness theorems.  $\Delta x = 0$ , "nothing", is the decisive criterion of a revised analysis, i.e. the differential and integral calculus.

The reimplementation of "nothing" initiates a re-evolution of the foundations of mathematics. Philosophical and mathematical Platonism are refuted by Nothing", "Nothing", nothing and "nothing".

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#### 1 Introduction

Nothing and emptiness are subject of countless writings of the cultures of mankind. Here the importance for mathematics is discussed. The millennia-old debate about the meaning of zero, with the result of its misinterpretation today, dates back to profound philosophical and religious differences in the metaphysical interpretation of Nothing and emptiness in the East and in the West and a misunderstanding of the meaning of "nothing" in mathematics.

### 2. History of 'nothing' in the different regions

- **2.1.** In Babylonia, in the 2nd millennium B.C., an empty place in the sequence of digits of a place value number indicated that there is no digit. Thus the concept of "zero" was invented, there was nothing. A millennium later the empty place, nothing, was marked to avoid mistakes. The sign for this nothing has the meaning "nothing". **The sign "zero" stands for "nothing" of digits and numbers, "no digit" and "no number" at that place.**
- **2.2.** The zero was known to the Greeks through their contacts with the Babylonians as marking an empty space. They associated it with the metaphysical Nothing. For Aristotle, this Nothing and emptiness are important themes, they are synonymous terms meaning non-existence. "Emptiness does not belong to the being". He also establishes the relation to zero. "Emptiness has no relation to things, just as Nothing has no relation to numbers". **The Greeks assumed that the zero symbolized the non-existent Nothing and therefore rejected it.** They rightly denied the existence of "number zero", but failed to recognize that the sign "zero" stands for existing "nothing" and indicates that no number exists at its place.
- **2.3.** Emptiness is important in Hinduism, but only in Buddhism it is focused on the center of philosophy. Sunyata, emptiness, becomes the comprehensive concept of ultimate cognition in Mahayana- Buddhism. It developed at the same time as the 0 was invented (the Babylonians had

<sup>1</sup> The topic including the bibliographical references are part of the book by Gert Treiber, "Nichts", Krise und reEvolution der Grundlagen der Mathematik, Cuvillier Verlag, 2020.

<sup>2</sup> Article I Empirical Fact: 0 is not a Number but represents "nothing"

<sup>3</sup> Article IV Inconsistency of the empty and the transfinite Set

used a sexagesimal system with the base 60 and two wedges for zero). Space is a metaphor for sunyata, emptiness. The 0 denotes an empty space, it is sunya, empty, "nothing" and is metaphysically connoted positive.

- **2.4.** In the course of their conquests, the Muslims adopted the 0 from the Indians meaning "sifr" for empty, "nothing" and brought it to Europe with this meaning.
- **2.5.** Leonardo of Pisa translated "sifr", "nothing", by the Latin "zephirum". However, by the reception of Aristotle the 0 was associated with the metaphysical Nothing and rejected. The resistance was intensified by the Christian religion. The church father Augustine assumes that God created the world out of Nothing. Consequently, Nothing is the opposite of the divine nature, the absence of God. Nothing is associated with evil, even with the devil. The 0 and the decimal system therefore were rejected in Europe for centuries. Only the contradictory definition of 0 to be something, digit and number, led to it being accepted.

# 3. Nothing and "Nothing"

A discussion about the existence of 'Nothing' expressing uncertainty and doubts had been decided by Plato in *Sophistes*: "Non-being was and is non-being and is to be counted as a notion among the many beings". Nothing does not exist, but the notion "Nothing" does exist. Metaphysical "Nothing" expresses absolute non-existence, while mathematical "nothing" means something very different.

# 4. nothing and "nothing"

There is nothing. This 'nothing' means that something that exists is not present at a certain place. Digits and numbers exist, but they are not present at an empty place. A gap in a sequence of digits is nothing, the designation of this empty space, the 0, represents "nothing". The conclusion that, as in the case of existing "Nothing," there exists a "nothing" to justify the zero, was not drawn by the Greeks, and therefore the zero was rejected as described above.

The "nothing" of sets is represented by the sign  $\emptyset$ . In Article IV it is demonstrated that the postulate of the axiomatic system of set theory, ZFC, the empty set  $\emptyset$  would exist as an element of the set, is false

- (1) **Definition:**  $\emptyset$  is the sign for "nothing" of sets, "no element" and "no set" In Article I it is demonstrated that 0 is not a number.
- (2) Definition: 0 is the sign for "nothing" of numbers, "no number"

A proof is a sequence  $\Gamma$  of formulas ( the sequent ), the last of them is the proven theorem. If a proposition is not provable, no such sequent exists. There is nothing in this regard. The 0 also characterizes this fact, it represents not only the "nothing" of numbers but also the "nothing" of proof theory of numbers.

- (3) Definition: 0 is the sign for "nothing" of proof theory, "no proof"
- (4) In article VI it is demonstrated that  $\Delta x = 0$ , "nothing", is the decisive criterion of a revised analysis, i.e. the differential and integral calculus.

## 5. Axioms of "nothing"

"Non-existence" and "nothing" are synonymous notions that allow equivalent propositions:

The **axiom of the "nothing" of sets** is formulated by (4). Linguistically applies:

The existence of the sign  $\emptyset$ , "nothing" of sets, is equivalent to the "non-existence" of a set M equal to  $\emptyset$ .. A sign is designated by  $\mathcal{Z}$ .

(5) Axiom:  $\exists \mathbb{Z}: \mathbb{Z} = \emptyset \leftrightarrow \neg (\exists M: M = \emptyset)$ 

If the set consists of numbers z, the **proposition of the "nothing" of numbers** is obtained (6):

The existence of the sign 0 is equivalent to non-existence of a number z equal to 0.

(6) Proposition:  $\exists \mathcal{Z}: \mathcal{Z} = 0 \leftrightarrow \neg (\exists z: z = 0)$ 

If the set consists of a sequence  $\Gamma$  the **proposition of "nothing" of proof theory** results:

The existence of the sign 0 is equivalent to the non-existence of a sequent  $\Gamma$  equal to 0.

(7) Proposition:  $\exists \mathcal{Z}: \mathcal{Z} = 0 \leftrightarrow \neg (\exists \Gamma: \Gamma = 0)$ 

Axiom (5) and propositions (6), (7) are also defined as mutual proof. For the proposition (7) this is realized because of its importance for the proof of Gödel's theorem G. Unprovable propositions are denoted by  $\varphi_{nb}$ .  $\Gamma = 0$  holds for the proof of  $\varphi_{nb}$ . Thus the axiom of "nothing" of number theory (8) follows from (7):

The existence of the sign 0 proves non-existence of a sequent  $\Gamma$  that would prove a proposition  $\phi_{nb}$ .

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(8) Axiom: \exists \mathcal{Z}: \mathcal{Z} = 0 \vdash \neg (\exists \Gamma: \Gamma \vdash \varphi_{nb})
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The propositions  $\varphi_{nb}$  include all false propositions as well as Gödel's true proposition G. Proof by nothing is impossible, but the existence of the sign 0 for "nothing", "no sequence", has evidentiary value. The axiom leads to the proof of Gödel's supposedly unprovable proposition and the exceedance of the incompleteness theorems (Article VII).

# 6. Refutation of Platonism by Nothing, "Nothing", nothing and "nothing"

Mathematical Platonism demands that mathematical objects and propositions exist detached from the material world and independent of human mind. Mathematics is not invented but discovered. But 'nothing' does not exist. The nothing of the empty space and the marking by "nothing", the **0**, are inventions of the human mind and do not exist independently of it. Mathematical Platonism is contradicted by nothing and "nothing".

Plato's theory of ideas demands that ideas, denoted by general concepts, own independent, superordinate existence. In **Plato**, each of these concepts must have an archetype in the world of ideas. But he **denied the existence of Nothing**, which consequently is not a prototype. But he demanded the existence of the notion "Nothing", which therefore **has no archetype**. It is pure conception of man. The "Nothing", the "nonexistent", refutes Plato's theory of ideas of being. **Philosophical and mathematical Platonism are refuted by Nothing**, "Nothing", nothing and "nothing".