I Empirical fact: 0 is not a number but represents "nothing"

Dec. 2024. In science today, 0 is undisputedly considered to be a number. But 0 marks an empty place in the sequence of digits of place value numbers. This empirical fact necessarily presupposes the meaning "nothing" of numbers, i.e. "no number". This statement was valid for three thousand years. In the 16th century, due to philosophical and religious reservations about the metaphysical Nothing, it was contradictorily reversed into its opposite, "something" and "number", thereby violating the tertium non datur. This wrong rationalistic definition is stipulated axiomatically. However, it shows no effects on applied mathematics, but causes contradictions in set theory.[1] The axiomatic re-implementation of 0 to represent "nothing" initiates a re-evolution of the foundations of mathematics.

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1. Introduction

Existence or non-existence of 'nothing' was a crucial issue in philosophy and religion of cultures in antiquity and later ages. When the zero appeared in mathematics in the 18th century BC in Mesopotamia, the connection with 'nothing' was inherent. But the different belief regarding existence of 'nothing' in diverse regions implied differing acceptance of zero meaning "nothing". The essential distinction of Nothing, nothing, "Nothing" and "nothing" was not achieved. The repercussions on mathematics are scrutinized in the following.

2. Nothing and nothing, "Nothing" and "nothing"

The still lasting confusion about the meaning of 0 originates in the insufficient distinction of the notions of the title of this section. Nothing with capital N means metaphysical absolute non-existence. This idea differs fundamentally from nothing with a small letter n, indicating that something that exists is not available at a certain place, there is nothing. The meaning of the quotation marks is explained by citing Plato. **Nothing does not exist, but the notion "Nothing" does exist.** The nothing with small letter n as well does not exist, but the notion "nothing" exists. **The empty place in a sequence of elememts is nothing, the mark of the void represents "nothing".** Aristotle, commenting on zero, however did not differentiate the notions appropriately. He described Nothing truly as non-existent and therefore rejected the zero, "nothing", as well, equating Nothing and "nothing". The zero was known in Greece through contacts with the Babylonians, but Greeks refused to use the sign because of this misconception. The denial of Nothing and "nothing" exerted repercussions in European Middle Ages.
The true meaning and the historical development of 0 are discussed below.

3. The 0 represents "nothing" of the digits and numbers, i.e. "no digit" and "no number"

The 0 marks an empty place in the sequence of digits of a place value number, e.g. 101 was initially written as 1 1 without the 0. This empirical fact is recognized in the mathematical literature,

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1 The topic including the bibliographical references are part of the book by Gert Treiber, *" Nichts", Krise und reEvolution der Grundlagen der Mathematik,* Cuvillier Verlag 2020.

but the consequences have not been considered.

In order to work out the meaning of the 0, an empty position in an arbitrary sequence of elements E is first considered (1). It is marked by an identifier K (2).

(1) E E E   E E E

(2) E E E K E E E

K cannot be an element E (3), otherwise the empty space would not be marked but filled.

(3) K ≠ E

The mark of an empty place in a sequence of elements represents "nothing" of the elements, i.e. "no element".

This also applies to a sequence of digits Z and the 0 (4). It cannot be a digit (5), otherwise the space would be filled, but represents "nothing" of the digits, "no digit".

(4) ZZZ0ZZZZ

(5) 0 ≠ Z

A digit is also a number. The 0 is therefore also "no number". This applies not only to the 0 within a place value number, but as well to the 0 as a term of equations (6), (7):

(6) $0 \pm n = \pm n$

Deletion of ±n on both sides of the equation gives:

(7) 0   =

The right side of the equation is left blank. **The 0 is also equal to an empty place, "nothing", in equations,** just as it is in a place value number.

The 0 is not a number, but represents "nothing" of digits and numbers, i.e. "no digit" and "no number".

Giuseppe Peano, on the other hand, axiomatically defined 0 as a natural number in 1902, with the meaning postulated today. This determination contradicts one of the fundamental axioms of logic, the 'tertium non datur': Either a statement A is true, or its negation, there is no third.

The meaning of the 0, "nothing" and "no number", is an empirically proven fact, so it is true.

The tertium non datur refutes the axiom that 0 is "something", a "number".

4. History of zero

4.1. The origin of zero

The first number systems did not have a zero, it was not needed for counting and arithmetic. The zero was only useful for place value numbers. The first of these systems was developed in Mesopotamia in the 2nd millennium BC. The blank space was unmarked for a millennium, 101 was written as 1 1.

The Babylonians used the base 60 of a sexagesimal system. The base 10 decimal system was invented by the Indians in the 3rd century AD. In 1150 **Bhaskara** stated in his *Lilavati* for the digits 1, 2, 3, … 9 and the 0:

"The place where none of the nine digits is present, is marked by a gap identified by a circle to avoid mistakes."

The meaning of the 0 is also expressed in the designation "sunya", for "empty", "nothing".

The Indians already calculated with the 0, but never in the meaning "number", always as "sunya". The assertion that is often made today, that calculating with 0 would justify it as a number, is wrong as (6) and (7) also show. **The 0 represents "no number" as well.**

The Arabs also adopted the 0 in the meaning of the Hindu system as 'sifr', 'empty', 'nothing'.

4.2. The 0 in Europe
4.2.1 The 0 means "nothing", "no digit" and "no number" for centuries

In the 10th century the Muslims brought the 0 meaning "nothing" to Europe. In his *liber abaci* at the beginning of the 13th century, Fibonacci translated the Arabic "sifr", "nothing", into the Latin "zephirum". **As late as the 16th century, the 0 was described as "nulla figura", "no digit" and "no number".** However, the 0 meaning "nothing" was burdened with a heavy prejudice.

In Europe, the metaphysical Nothing, to which the mathematical "nothing" was untruly equated, was banned; the Greeks of antiquity had already brought about this. Aristotle described the Nothing as non-existent and therefore also rejected the zero, which was known in Greece through contacts with the Babylonians. In India, on the other hand, there were no reservations, "nothing" had positive connotations.

In the religiously influenced Middle Ages, Nothing moreover was ostracized by the church father Augustine. For him, Nothing meant the absence of God and was associated with the devil.

The decimal system was therefore unable to assert itself in Europe for centuries against the inferior Roman system. In the 16th century, however, the advantage became more and more serious due to the emerging bookkeeping and the decimal system with the 0 was finally adopted.

That demanded its price.

4.2.2. In the 16th century 0 inconsistently becomes "something", "digit"and "number"
The devilish "nothing" and "no number" contradictoryly became "something" and "number" in a confused upheaval that lasted many decades.

Some mathematicians such as Descartes, Leibniz and Euler still maintained the meaning of "nothing" but no longer doubted the "number". The fact that "nothing" of the numbers cannot be a number was not recognized. But then it gradually disappeared from mathematics, "nothing" is not defined today.

Peano then created the axiom in 1902 that establishes 0 as a natural number.

Mathematicians, familiar with the history of 0, know that originally it represented "nothing" and "no number". Nevertheless, they do not doubt that 0 is rightly defined as a number. After all, irrational and imaginary quantities such as $\sqrt{2}$ and $\sqrt{-1}$ were initially not accepted as numbers either, but were then included among them. The original meaning of the 0, "nothing", were just an irrelevant historical relict.

But irrational and imaginary quantities were always "something" that could be interpreted as "something else" without contradiction. The inherent contradiction in reversing the 0 from "nothing" and "no number" to "something" and "number" was not recognized.

The historical development of the meaning of the 0 shows that the "number 0" cannot be justified on the basis of the timeless facts listed.

It is the result of metaphysical reservations about the actual meaning of "nothing".

The wrong definition didn't have any effect for a long time, since the alleged "number 0" was and is still ascribed the properties of "nothing". This only changed in set theory and the contradiction became virulent. It is not surprising that the antipode of zero, infinity, is affected, as is demonstrated in article II.

5. The axiom of "nothing" of numbers

The 0 is again defined according to its true meaning:
(8) Definition: The 0 is the sign for "nothing" of numbers, i.e. "no digit" and "no number".
A sign is marked by $\mathcal{S}$, numbers by x, thus formulating the axiom of "nothing" of numbers":
(9) Axiom: $\exists\, \mathcal{S}: \mathcal{S} = 0 \leftrightarrow \forall\, x: \neg\, (\, \exists\, x : x = 0\,)$

The sign $\mathcal{S} = 0$ exists, meaning "no digit" and "no number", equivalent to this is the proposition that all numbers x are not equal to 0.
The axiom can also be formulated as a mutual proof of the two propositions.

The reimplementation of "nothing" initiates a re-evolution of the foundation of mathematics.
The axiom of "nothing" of numbers disproves the existence of transfinite numbers (article II).
The axiom of "nothing" of sets disproves the existence of transfinite sets (article III).
The axiom of "nothing" of proof disproves the incompleteness of the theory of natural numbers (article VII).